



Tanta University



Faculty of Engineering

ELECTRICAL POWER ENGINEERING (1) DC DISTRIBUTION

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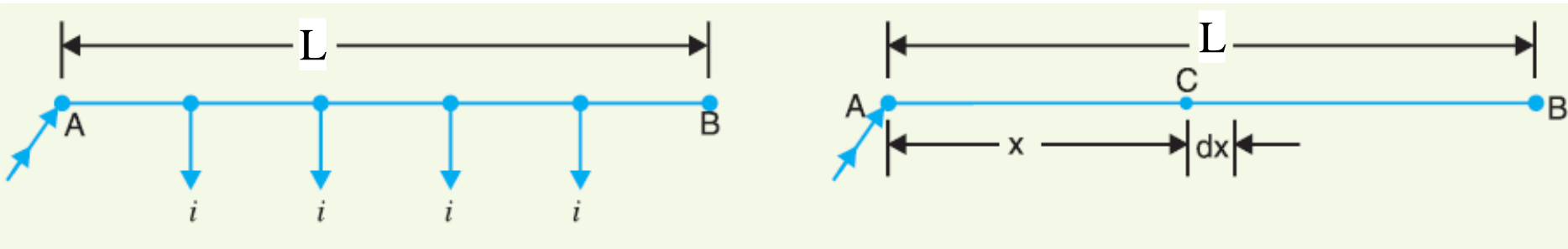
Department of Elec. Power and Machines Eng.

Uniformly Loaded Distributor Fed at One End

The 2-wire d.c. distributor AB is fed at one end A and loaded uniformly with i amperes per metre length

At every 1 m length of the distributor, the load tapped is i amperes

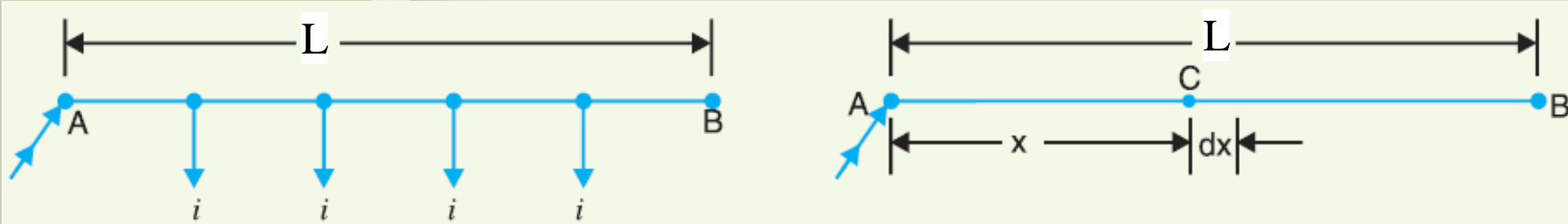
Let L meters be the length of the distributor and r ohm be the resistance per meter run.



Point C on the distributor at a distance “ x ” m from the feeding point A

Current at point C is $= i \cdot L - i \cdot x = i (L - x)$ A

Uniformly Loaded Distributor Fed at One End



For a small length dx , the resistance is $r \, dx$

The voltage drop over length dx is:

$$dv = i (L - x) r \, dx = i r (L - x) \, dx$$

Voltage drop to point C:

$$V = \int_0^x i r (L - x) \, dx = i r \left(Lx - \frac{x^2}{2} \right)$$

To get the voltage drop to point B, put $x = L$

Voltage drop for distributor AB = $i r \left(L \times L - \frac{L^2}{2} \right)$

Voltage drop for AB = $0.5 (i L)(r L) = 0.5 I R$

(similar to the same load concentrated at the middle point)

Uniformly Loaded Distributor Fed at One End

Example: A 2-wire d.c. 200m long distributor fed from one end is uniformly loaded with 2A/m. Resistance of single wire is 0.3 Ω /km. Calculate: (i) the voltage drop to 150 m from feeding point (ii) the max. voltage drop

Solution: Current loading, $i=2$ A/m

Resistance per meter run, $r=2 \times 0.3/1000 = 0.0006$ Ω /m

Length of distributor, $L=200$ m

(i) Voltage drop to distance x meter from feeding point

$$= i r \left(Lx - \frac{x^2}{2} \right)$$

$$\text{voltage drop} = 2 \times 0.0006 \left(200 \times 150 - \frac{150 \times 150}{2} \right) = \mathbf{22.5 \text{ V}}$$

Uniformly Loaded Distributor Fed at One End

(ii) Total current entering the distributor:

$$I = i \times L = 2 \times 200 = 400 \text{ A}$$

Total resistance of the distributor:

$$R = r \times L = 0.0006 \times 200 = 0.12 \Omega$$

Total drop over the distributor:

$$= 0.5 I R = 0.5 \times 400 \times 0.12 = 24 \text{ V}$$

Uniformly Loaded Distributor Fed at Both Ends

Distributor fed at both ends with equal voltages

Consider a distributor AB of length L meters, having resistance r ohms per meter run and with uniform loading of i amperes per meter run

Let the distributor be fed at the feeding points A and B at equal voltages, say V volts

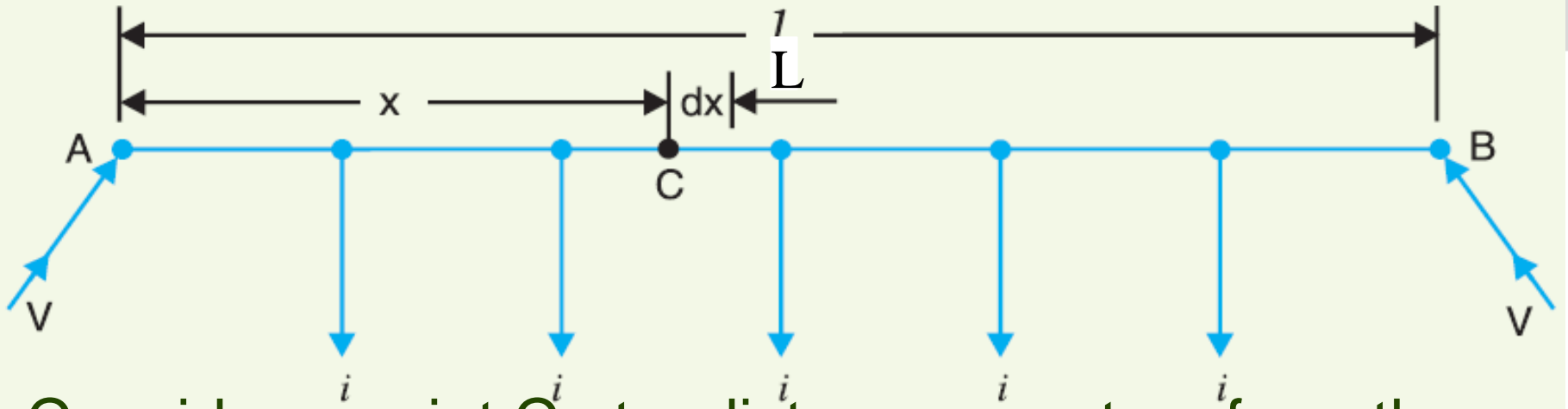
The total current supplied to the distributor is iL

As the two end voltages are equal, therefore, current supplied from each feeding point is: $iL / 2$

Current supplied from each feeding point: $= \frac{iL}{2}$

Uniformly Loaded Distributor Fed at Both Ends

Distributor fed at both ends with equal voltages



Consider a point C at a distance x meters from the feeding point A

Then current at point C is $= \frac{iL}{2} - iX = i\left(\frac{L}{2} - X\right)$

- $dV = i\left(\frac{L}{2} - x\right) r dx = ir\left(\frac{L}{2} - x\right) dx$
- **Voltage drop at point C** $= \int_0^x ir\left(\frac{L}{2} - x\right) dx = \frac{ir}{2}(Lx - x^2)$

Uniformly Loaded Distributor Fed at Both Ends

Distributor fed at both ends with equal voltages

Maximum voltage drop will occur at mid-point i.e. where $x = L/2$

$$\text{Max. voltage drop} = \frac{i r}{2} (Lx - x^2)$$

$$= \frac{i r}{2} \left(L \times \frac{L}{2} - \frac{L^2}{4} \right) = \frac{1}{8} i r L^2 = \frac{1}{8} (iL) (rL) = \frac{1}{8} I R$$

$$\text{Minimum voltage} = V - \frac{I R}{8} \text{ volts}$$

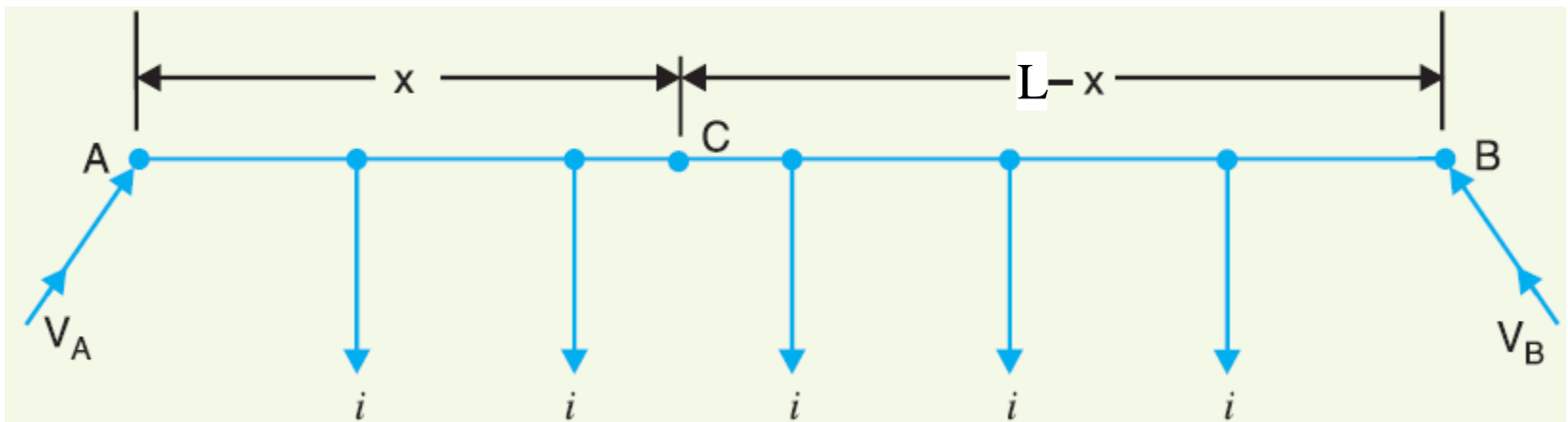
Distributor fed at both ends with unequal voltages

Consider a distributor AB of length L meters having resistance r ohms per meter run and with a uniform loading of i amperes per meter run

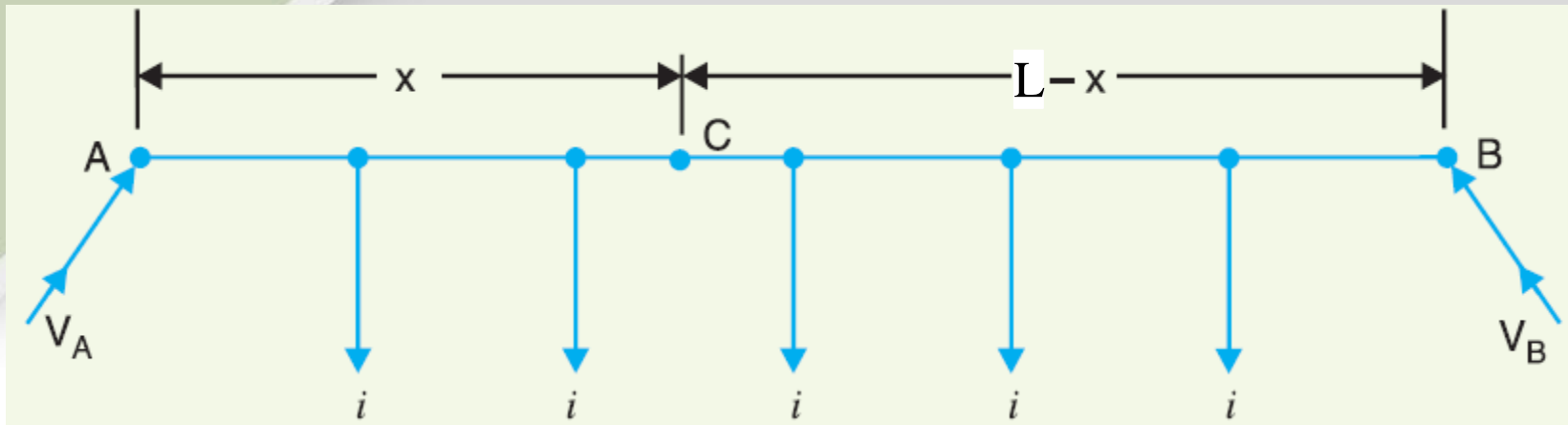
Let the distributor be fed from feeding points A and B at voltages V_A and V_B respectively

The point of minimum potential C is situated at a distance x meters from the feeding point A

The current supplied by the feeding point A will be $i x$



Distributor fed at both ends with unequal voltages



Voltage drop in section AC = $0.5 i r x^2$

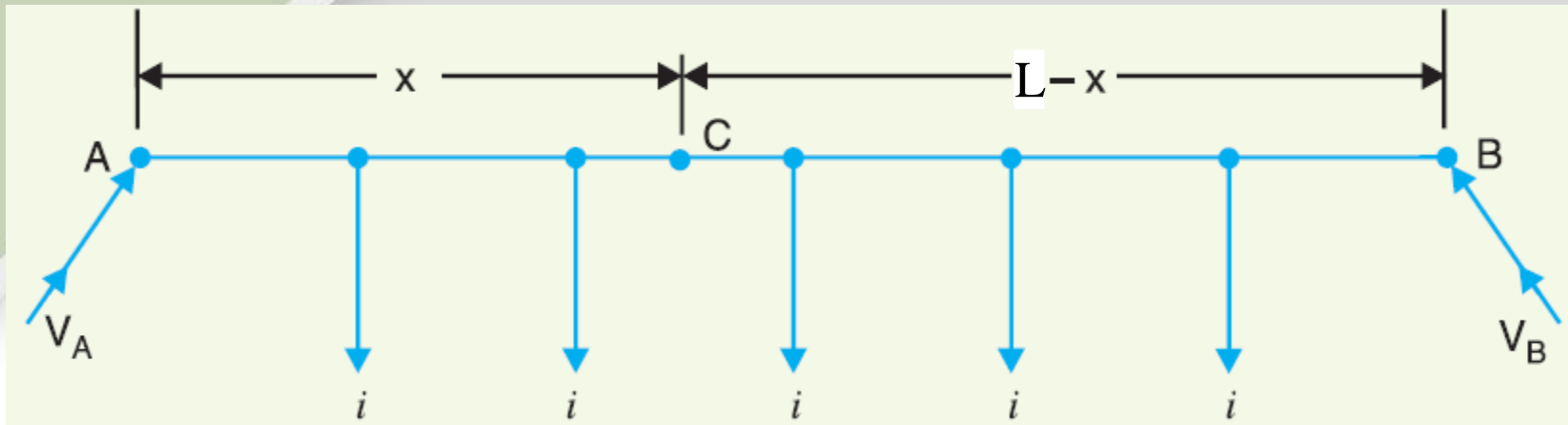
As the distance of C from feeding point B is $(L - x)$,
current fed from B is $i (L - x)$

Voltage drop in section BC = $\frac{i r (L - x)^2}{2}$ volts

Voltage at point C, V_C

$$= V_A - \text{Drop over AC} = V_A - \frac{i r x^2}{2}$$

Distributor fed at both ends with unequal voltages



Also, voltage at point C, V_C

$$= V_B - \text{Drop over BC} = V_B - \frac{i r (L-x)^2}{2}$$

$$V_A - \frac{i r x^2}{2} = V_B - \frac{i r (L-x)^2}{2}$$

Solving for x , we get,

$$x = \frac{V_A - V_B}{i r L} + \frac{L}{2}$$

Example: A two-wire d.c. distributor cable 1000 m long is loaded with 0.5 A/m. Resistance of each conductor is 0.05 Ω /km. Calculate the maximum voltage drop if the distributor is fed from both ends with equal voltages of 220V. What is the min. voltage and where it occurs?

Solution.

Resistance of distributor/m, $r = 2 \times 0.05/1000 = 0.1 \times 10^{-3} \Omega$

Length of distributor, $L = 1000 \text{ m}$

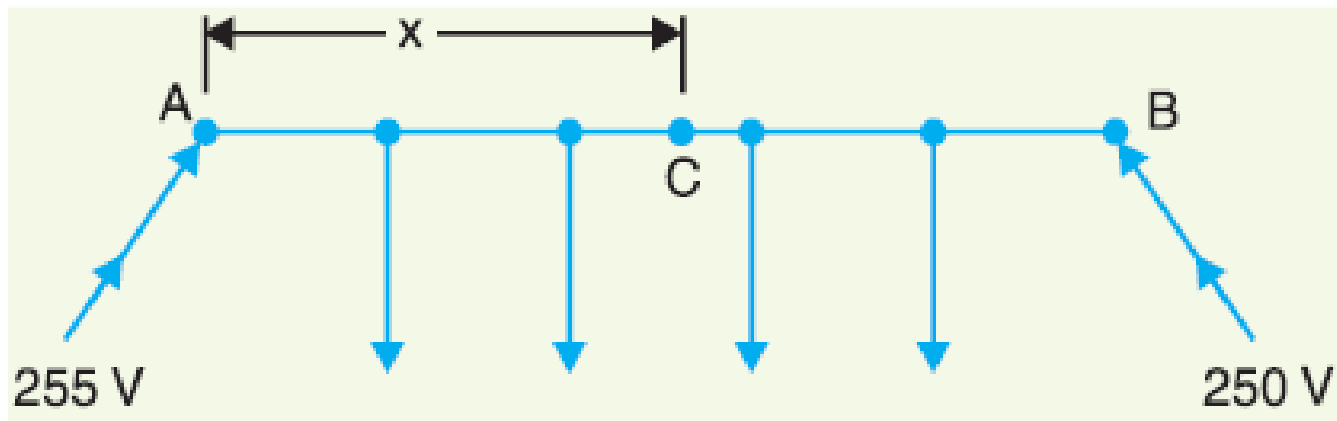
Total current supplied by distributor, $I = i L = 0.5 \times 1000 = 500 \text{ A}$

Total resistance of the distributor, $R = r L = 0.1 \times 10^{-3} \times 1000 = 0.1 \Omega$

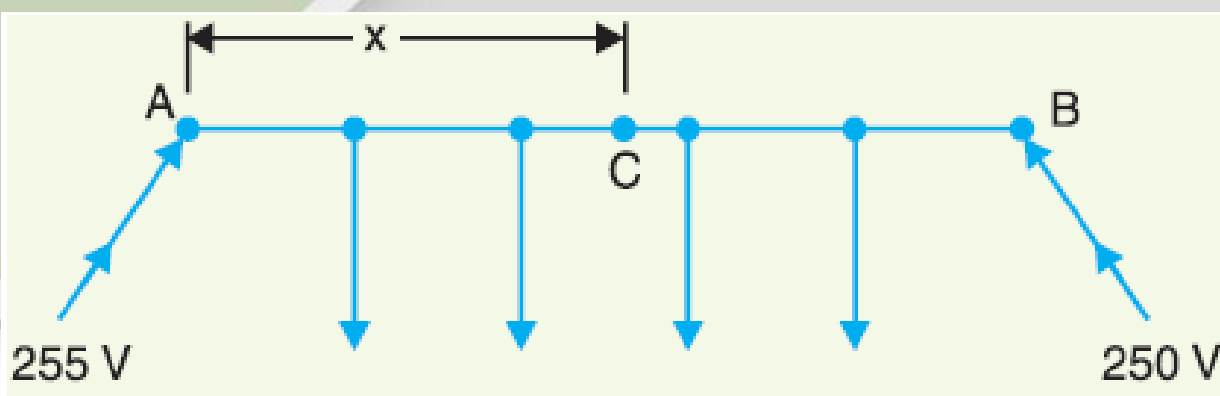
$$\therefore \text{Max. voltage drop} = \frac{I R}{8} = \frac{500 \times 0.1}{8} = \mathbf{6.25 \text{ V}}$$

Minimum voltage will occur at the mid-point of the distributor and its value is
 $= 220 - 6.25 = \mathbf{213.75 \text{ V}}$

Example: A 2-wire d.c. distributor AB 500 m long is fed from both ends and is loaded uniformly at the rate of 1.0 A/m. At feeding point A, the voltage is maintained at 255V and at B at 250V. If the resistance of each conductor is 0.1Ω per km, determine: (i) the minimum voltage and the point where it occurs (ii) the currents supplied from feeding points A and B



Solution: Resistance of distributor/m,
$$r = 2 \times 0.1/1000 = 0.0002 \Omega$$

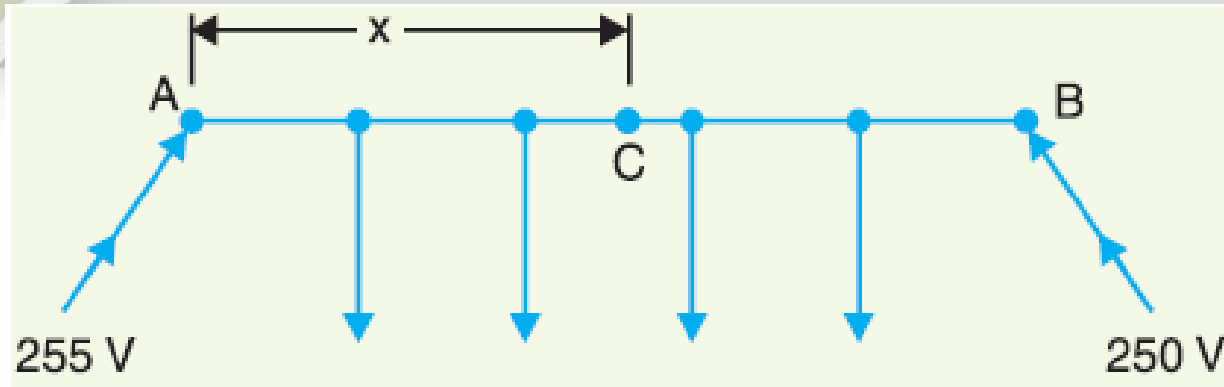


- (i) Let the minimum potential occur at a point C distant x metres from the feeding point A

$$\begin{aligned}
 x &= \frac{V_A - V_B}{i r L} + \frac{L}{2} = \frac{255 - 250}{1 \times 0.0002 \times 500} + 500 / 2 \\
 &= 50 + 250 = \mathbf{300 \text{ m}}
 \end{aligned}$$

i.e. minimum potential occurs at 300 m from point A.

$$\begin{aligned}
 V_C &= V_A - \frac{i r x^2}{2} = 255 - \frac{1 \times 0.0002 \times (300)^2}{2} \\
 &= 255 - 9 = \mathbf{246 \text{ V}}
 \end{aligned}$$



(ii) Current supplied from A = $i \times x = 1 \times 300 = 300 \text{ A}$

Current supplied from B = $i (L - x) = 1 (500 - 300) = 200 \text{ A}$

Report

Derive an expression for the **total power losses** and **the minimum voltage** in a uniformly loaded distributor:

1. **Feeding at one end**
2. **Feeding at both ends with equal voltages**
3. **Feeding at both ends with unequal voltages**

Ring Distributor

A distributor arranged to form a closed loop and fed at one or more points is called a **ring distributor**.

Such a distributor starts from one point, makes a loop through the area to be served, and returns to the original point

The distributor can be considered as consisting of a series of open distributors fed at both ends

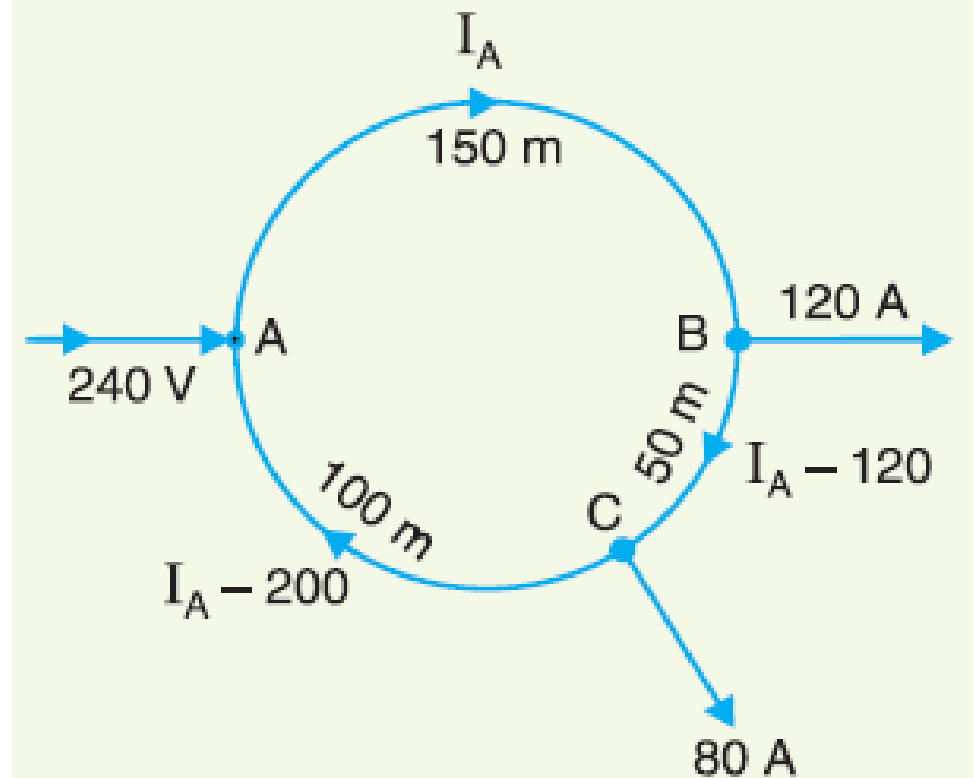
The principal advantage of ring distributor is that by proper choice in the number of feeding points, great economy in copper can be affected

Ring Distributor

Example: A 2-wire d.c. ring distributor is 300 m long and is fed at 240 V at point A. At point B, 150 m from A, a load of 120 A is taken and at C, 100 m in the opposite direction, a load of 80 A is taken.

If the resistance per 100 m of single conductor is $0.03\ \Omega$, find:

- (i) current in each section of distributor
- (ii) voltage at points B and C



Ring Distributor

Resistance per 100 m of distributor = $2 \times 0.03 = 0.06 \Omega$

Res. of section AB, $R_{AB} = 0.06 \times 150/100 = 0.09 \Omega$

$R_{BC} = 0.06 \times 50/100 = 0.03 \Omega$, $R_{CA} = 0.06 \times 100/100 = 0.06 \Omega$

(i) Let the current I_A flows in section AB. Then currents in sections BC and CA will be $(I_A - 120)$ and $(I_A - 200)$

The voltage drop in the loop ABCA is zero

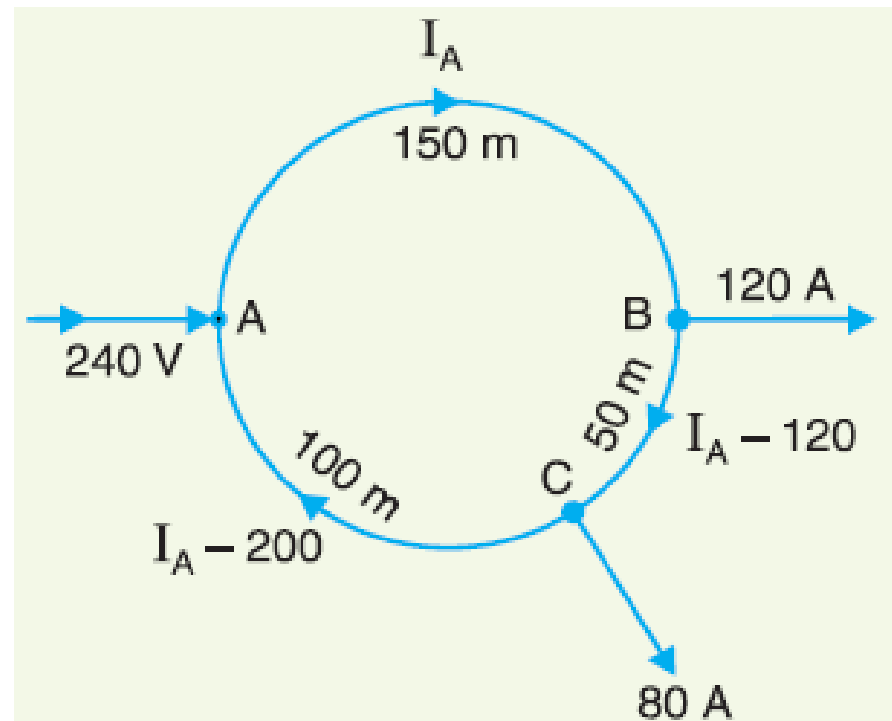
$$I_{AB} R_{AB} + I_{BC} R_{BC} + I_{CA} R_{CA} = 0$$

$$0.09 I_A + 0.03 (I_A - 120)$$

$$+ 0.06 (I_A - 200) = 0$$

$$0.18 I_A = 15.6$$

$$\therefore I_A = 15.6/0.18 = 86.67 \text{ A}$$



Ring Distributor

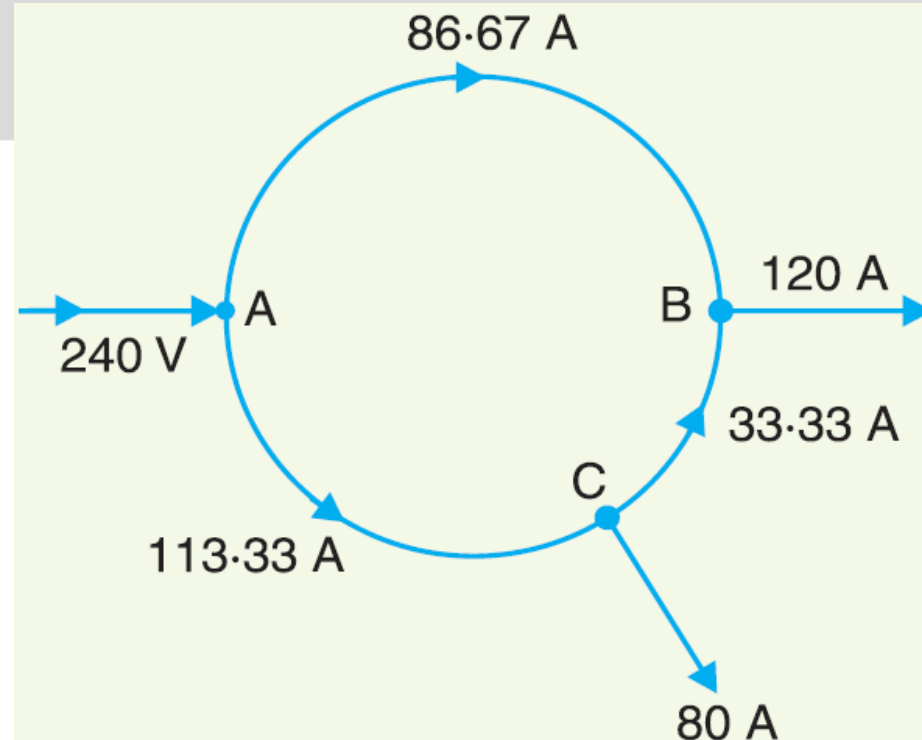
it is seen that B is the point of minimum potential

Current in section AB is

$$I_{AB} = I_A = 86.67 \text{ A from A to B}$$

$$I_{BC} = I_A - 120 = -33.33 \text{ A} = 33.33 \text{ A from C to B}$$

$$I_{CA} = I_A - 200 = -113.33 \text{ A} = 113.33 \text{ A from A to C}$$



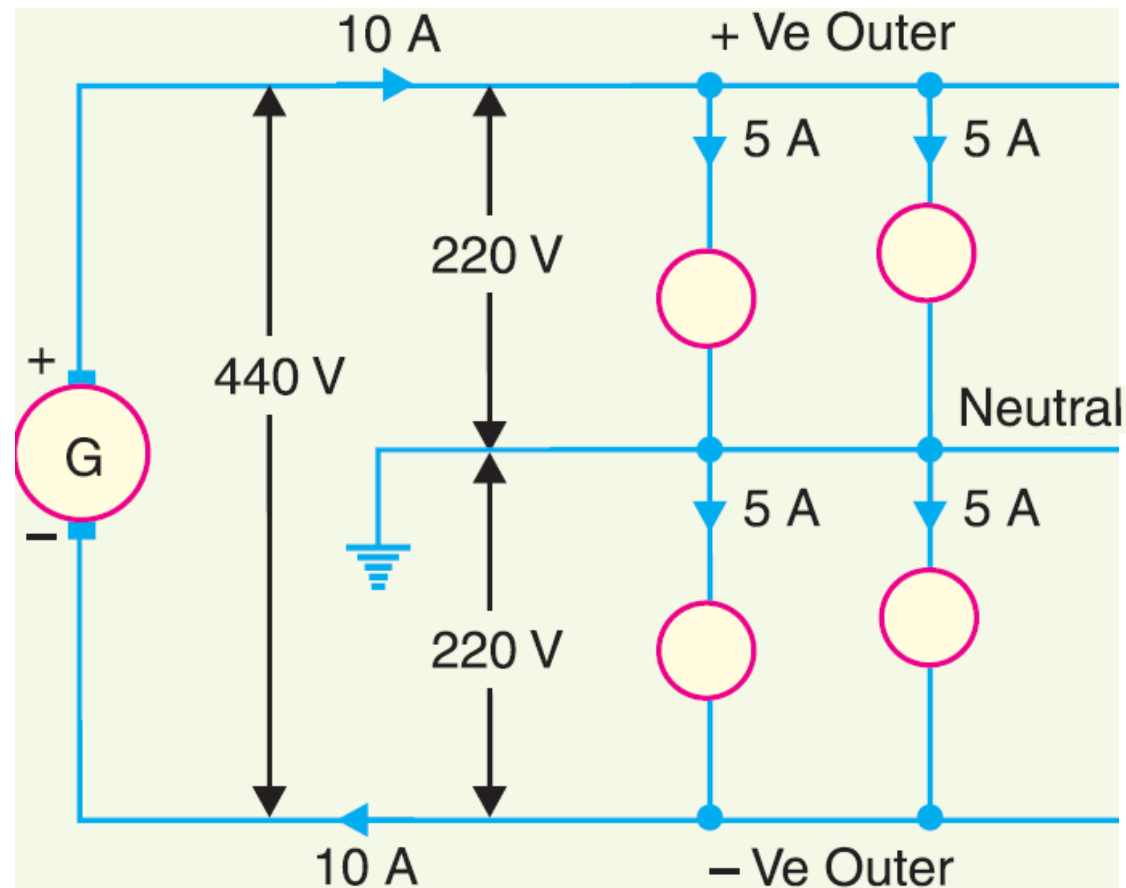
$$\begin{aligned} \text{(ii) Voltage at point B, } V_B &= V_A - I_{AB} R_{AB} \\ &= 240 - 86.67 \times 0.09 = 232.2 \text{ V} \end{aligned}$$

$$\begin{aligned} \text{Voltage at point C, } V_C &= V_B + I_{BC} R_{BC} \\ &= 232.2 + 33.33 \times 0.03 = 233.2 \text{ V} \end{aligned}$$

Three-Wire D.C. System

Three wire d.c. system can provide two voltages: V volts between any outer and neutral and $2V$ volts between the outers

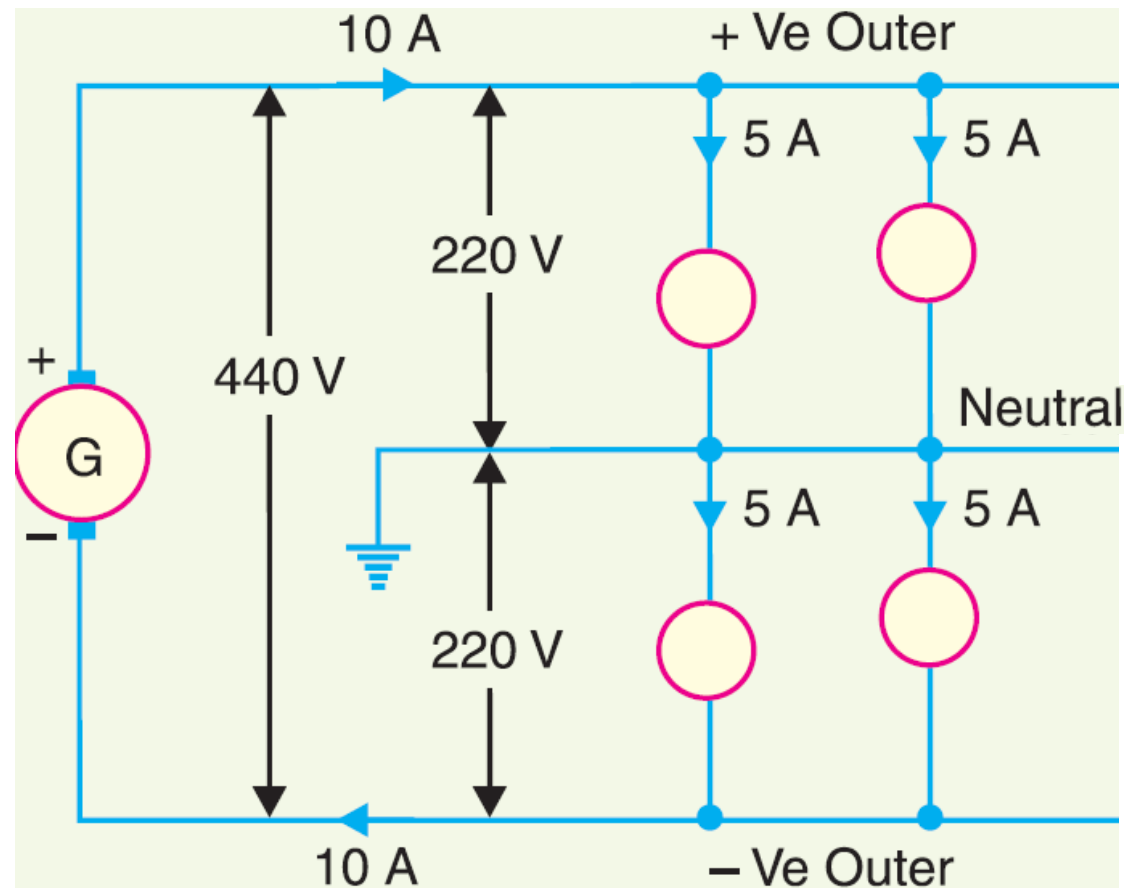
Motors needing high voltage are fixed between the outers where lighting and heating loads needing less voltage are connected between any one outer and the neutral



Three-Wire D.C. System

- (i) If the loads applied on both sides of the neutral are equal, the current in the neutral wire will be zero

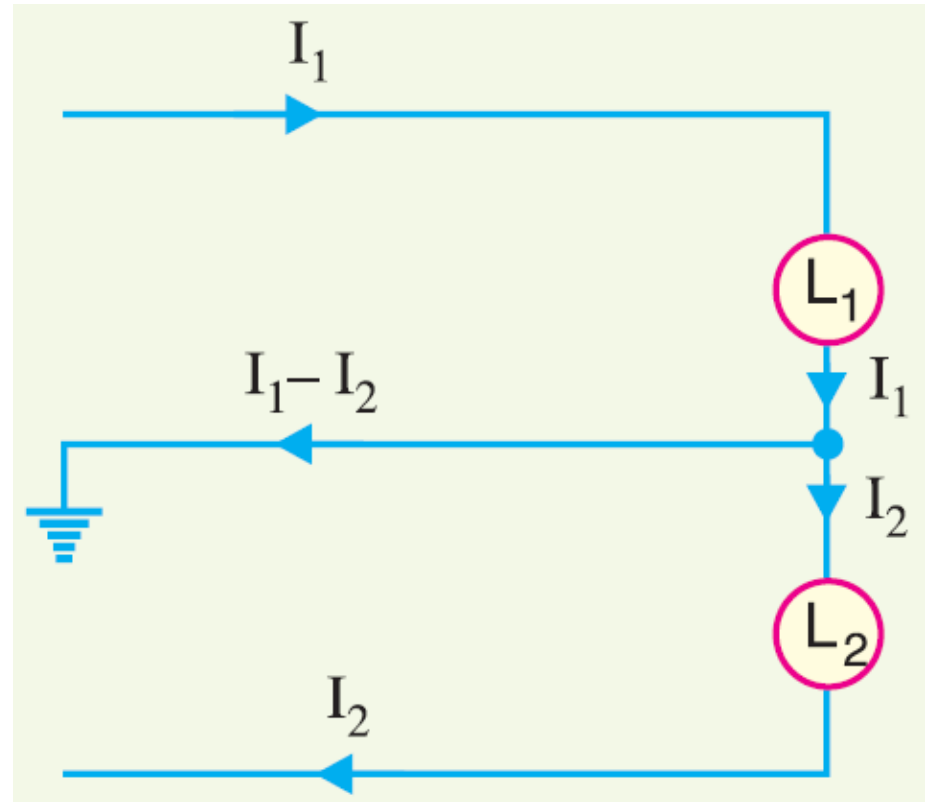
The potential of the neutral will be exactly half-way between the potential difference of the outers



Three-Wire D.C. System

(ii) If the load on the positive outer (I_1) is greater than on the negative outer (I_2), the difference ($I_1 - I_2$) will flow in the neutral wire from load end to supply end

The potential of neutral wire will no longer be midway between the potentials of the outers



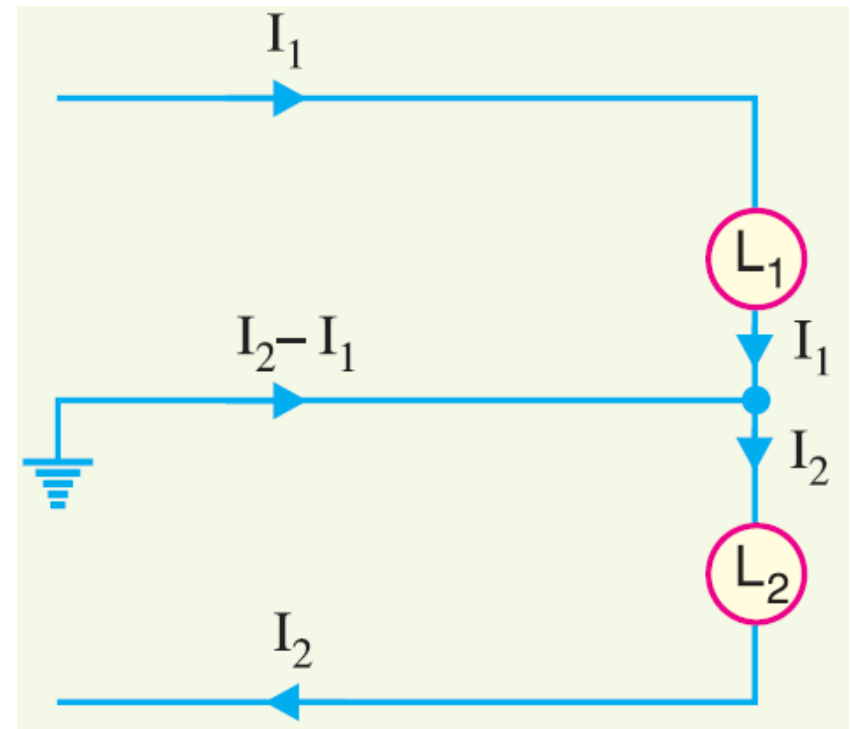
Three-Wire D.C. System

(iii) If the load on the negative outer (I_2) is greater than on the positive outer (I_1), the difference ($I_2 - I_1$) will flow in the neutral from supply end to load end

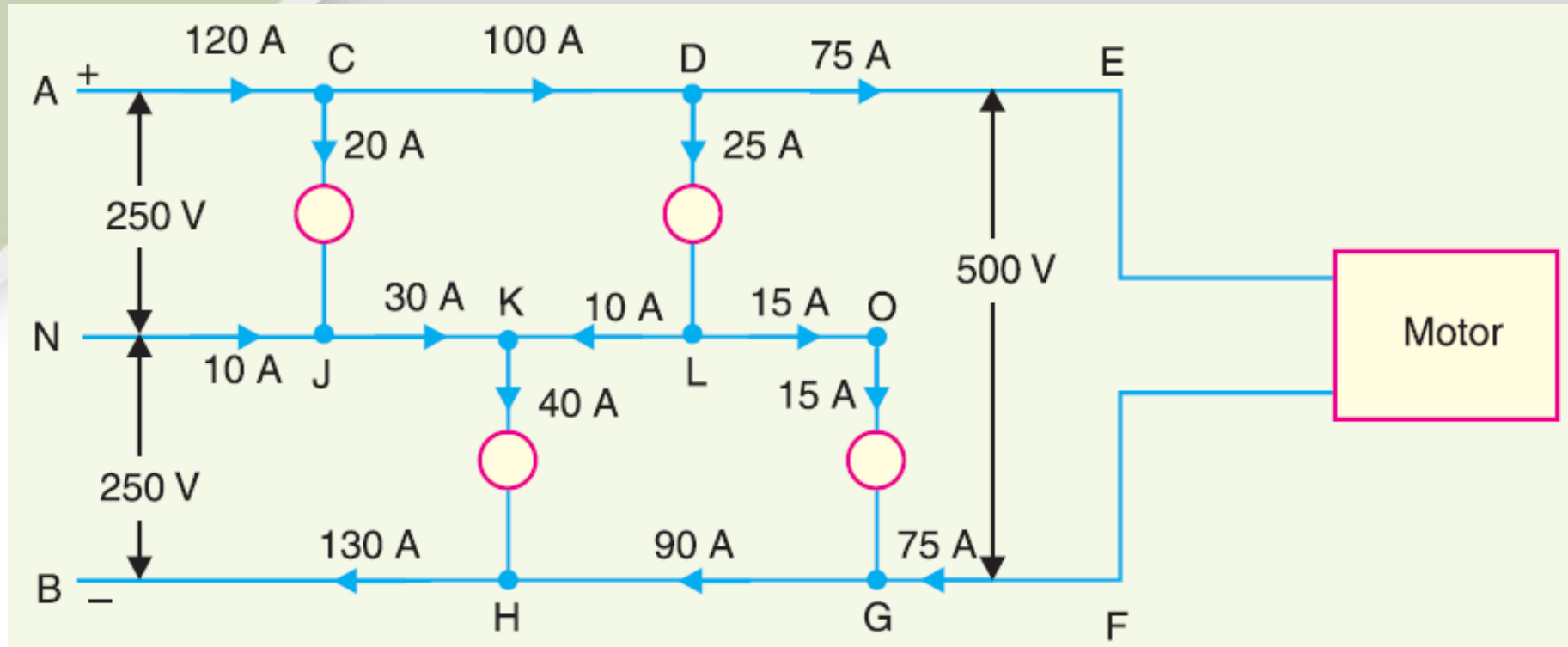
Again, the neutral potential will not remain half-way between that of the outers

It is desirable that voltage between any outer and the neutral should have the same value

This is achieved by distributing the loads equally on both sides of the neutral



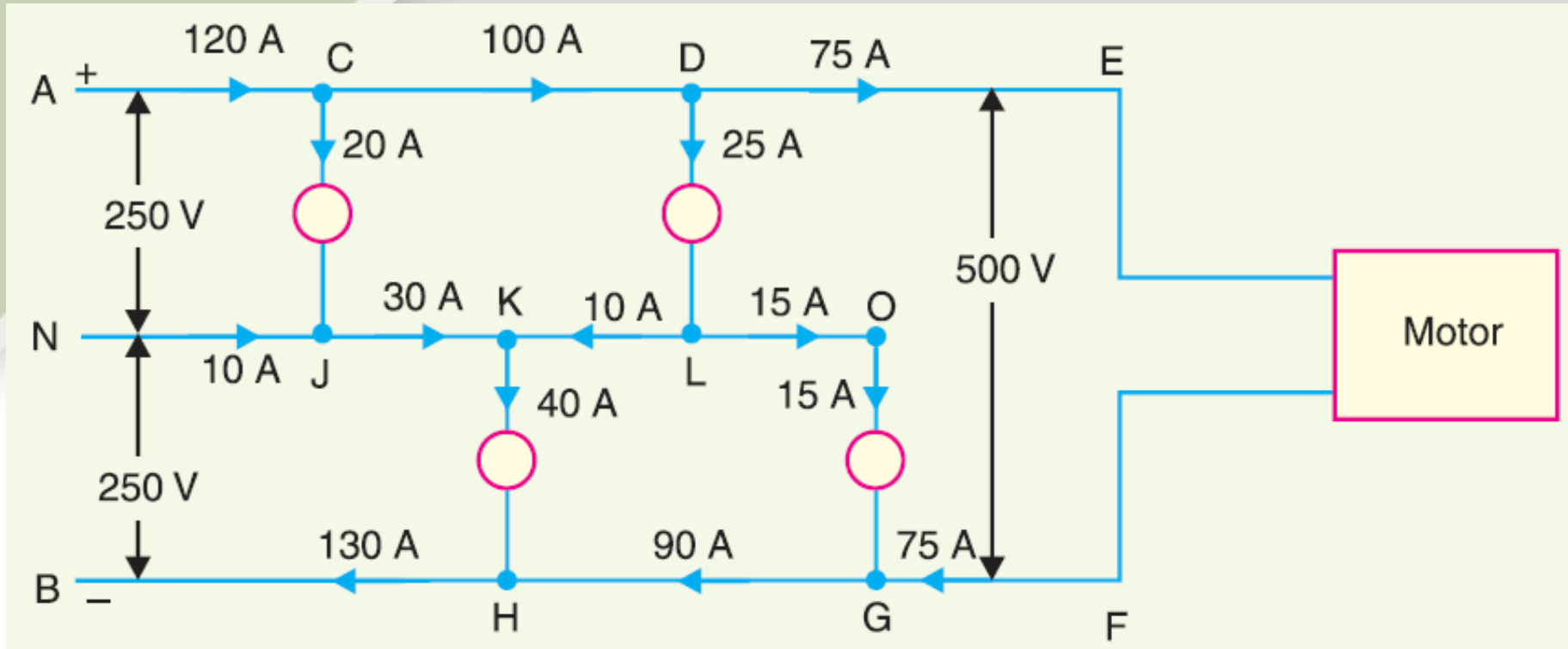
Current Distribution in 3-Wire D.C. System



It is clear that a current of 120 A enters the positive outer, while 130 A comes out of the negative outer

$130 - 120 = 10 \text{ A}$ must flow in the neutral at point N

Load-point voltages

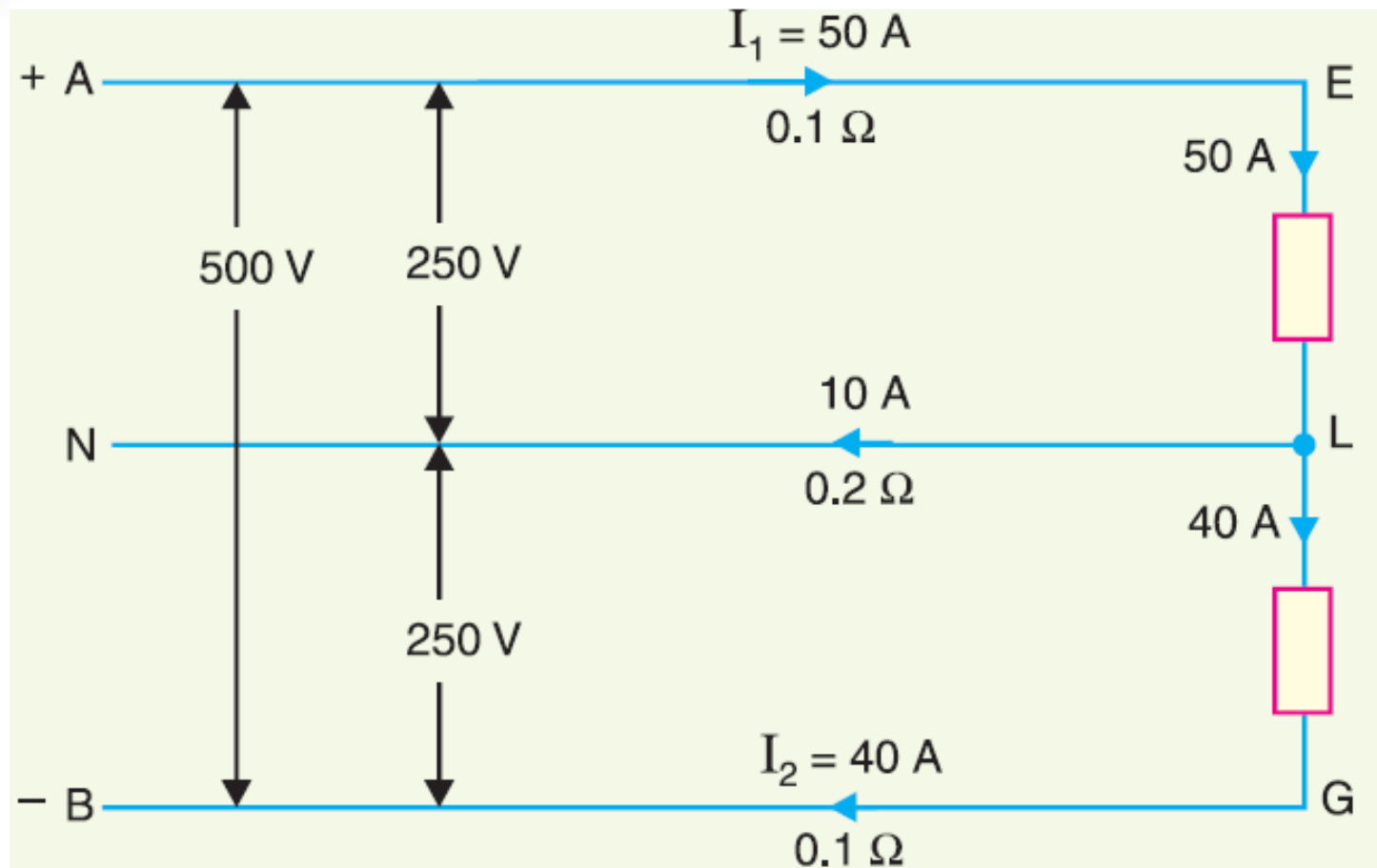


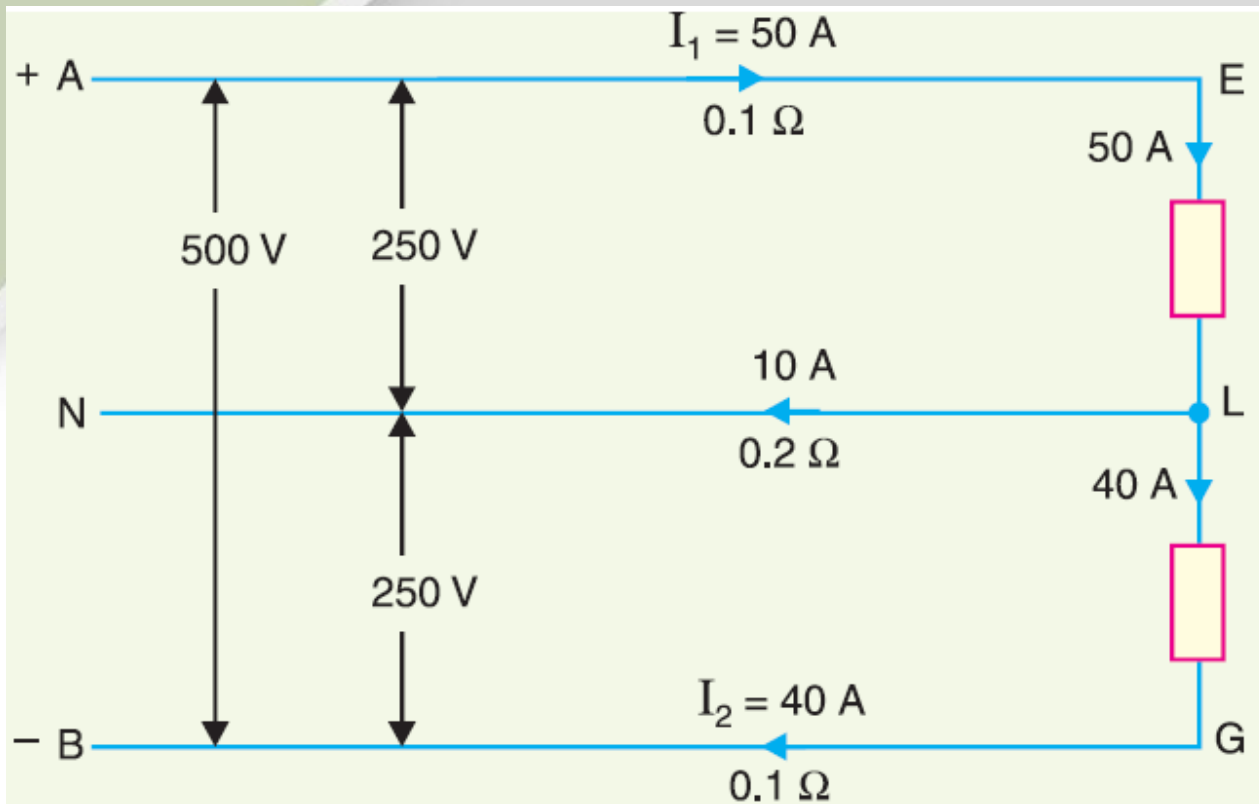
The voltage across load CJ is obtained by applying Kirchhoff's voltage law to the loop ACJNA

$$\text{Voltage across CJ} = 250 - \text{drop in AC} + \text{drop in NJ}$$

Example: A load supplied on 3-wire d.c. system takes a current of 50 A on the +ve side and 40 A on the -ve side. The resistance of each outer wire is 0.1Ω and the cross-section of middle wire is one-half of that of outer. If the system is supplied at 500/250 V, find the voltage at the load end between each outer and middle wire.

Example: A load supplied on 3-wire d.c. system takes a current of 50 A on the +ve side and 40 A on the -ve side. The resistance of each outer wire is $0.1\ \Omega$ and the cross-section of middle wire is one-half of that of outer. If the system is supplied at 500/250 V, find the voltage at the load end between each outer and middle wire.



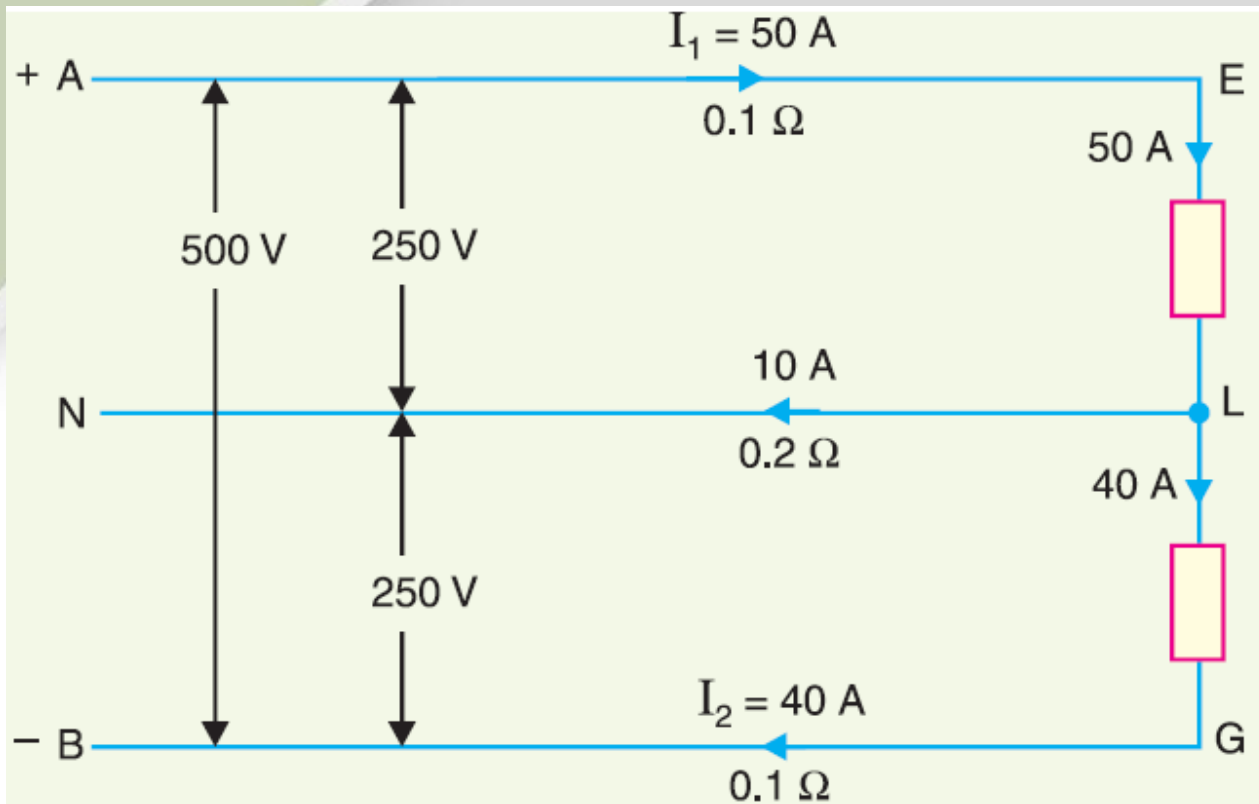


Current in the neutral wire is $50 - 40 = 10\text{A}$

As the X-sectional area of neutral is half that of outer, therefore, its resistance $= 2 \times 0.1 = 0.2 \Omega$

Voltage at the load end on the +ve side,

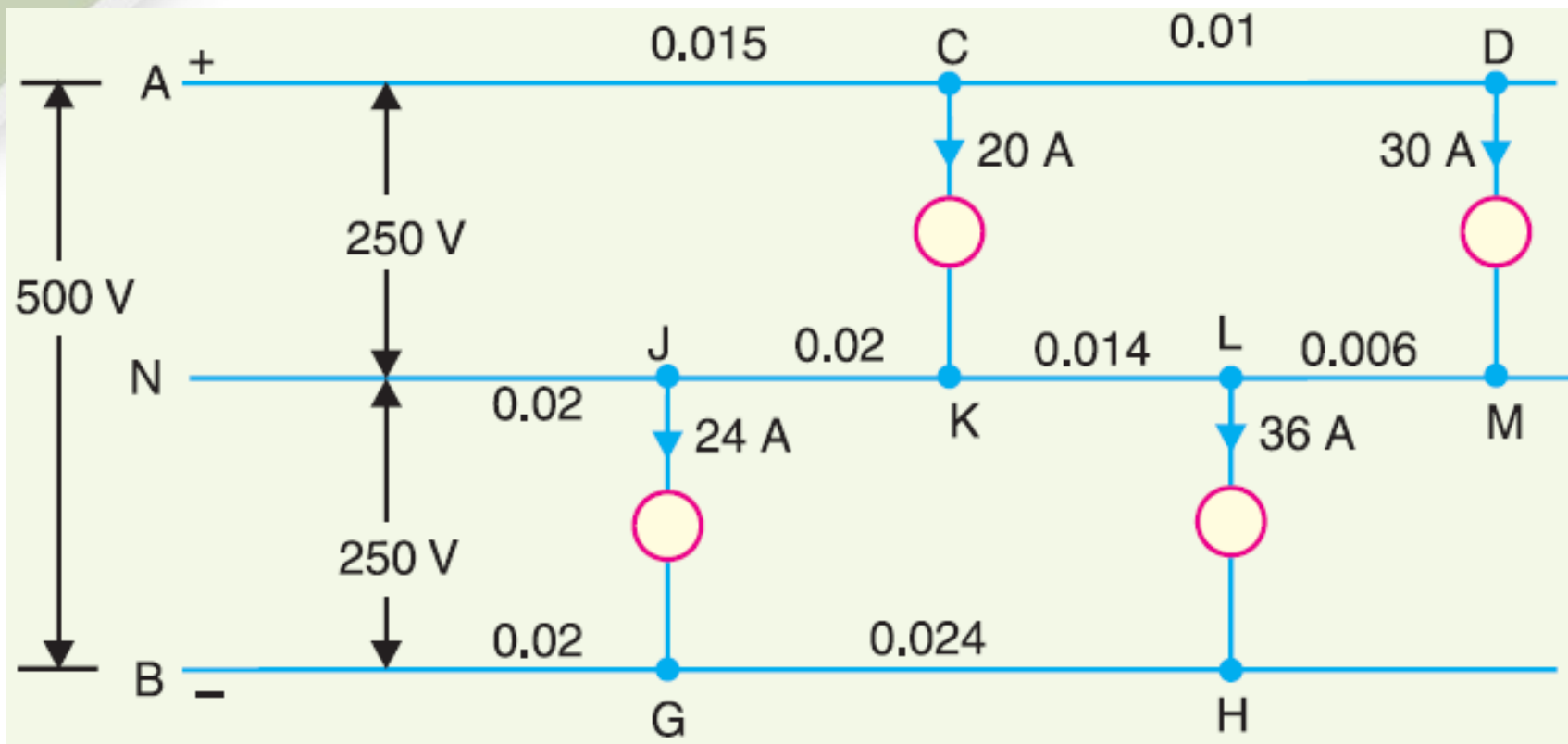
$$V_{EL} = 250 - I_1 R_{AE} - (I_1 - I_2) R_{NL} = 250 - 50 \times 0.1 - (10) \times 0.2 = 243\text{V}$$

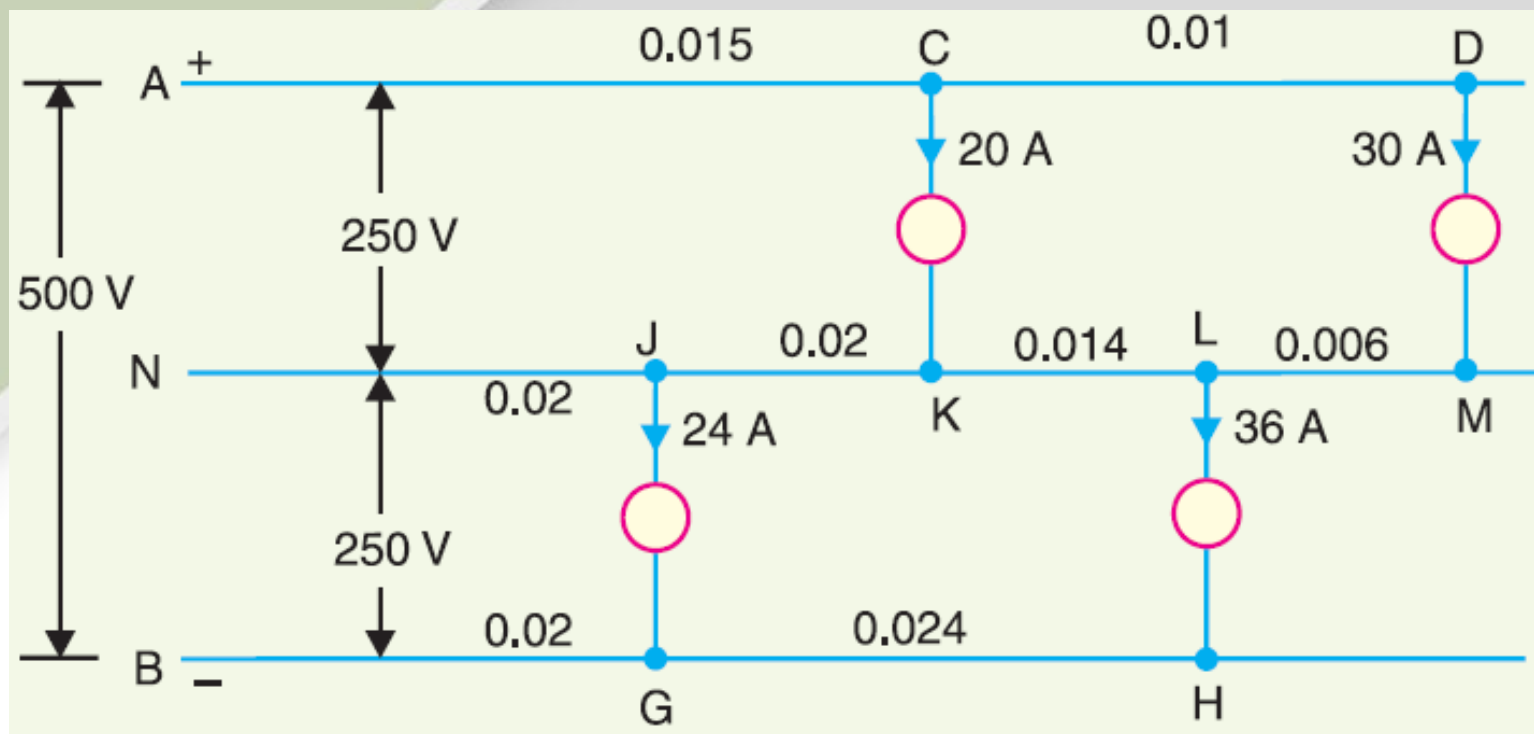


Voltage at the load end on the -ve side,

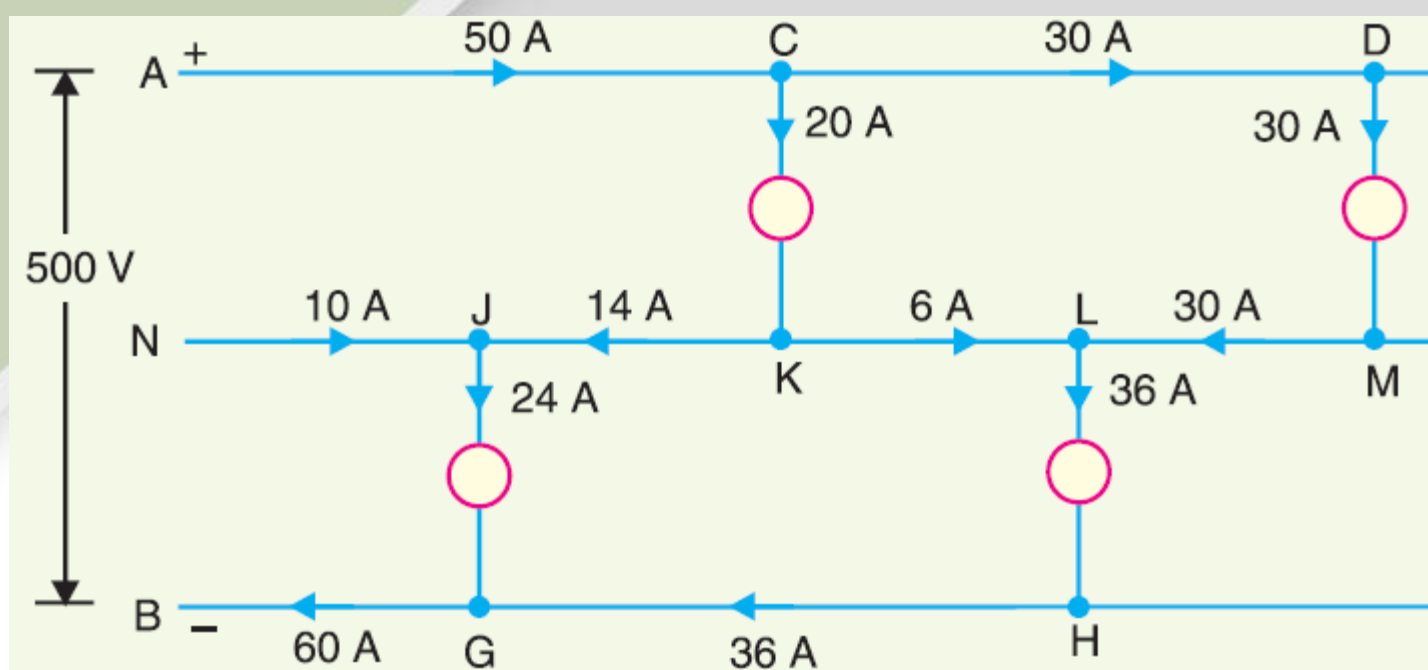
$$\begin{aligned}
 V_{LG} &= 250 + (I_1 - I_2) R_{NL} - I_2 R_{BG} \\
 &= 250 + 10 \times 0.2 - 40 \times 0.1 = 248\text{V}
 \end{aligned}$$

Example: A 3-wire, 500/250 V distributor is loaded as shown. The resistance of each section is given in ohm. Find the voltage across each load point.





Section	Resistance (Ω)	Current (A)	Drop (V)
AC	0.015	50	0.75
CD	0.01	30	0.3
ML	0.006	30	0.18
KL	0.014	6	0.084
KJ	0.02	14	0.28
NJ	0.02	10	0.2
HG	0.024	36	0.864
GB	0.02	60	1.2



Voltage across load $CK = 250 - \text{drop in } AC - \text{drop in } KJ + \text{drop in } NJ = 250 - 0.75 - 0.28 + 0.2 = \mathbf{249.17 \text{ V}}$

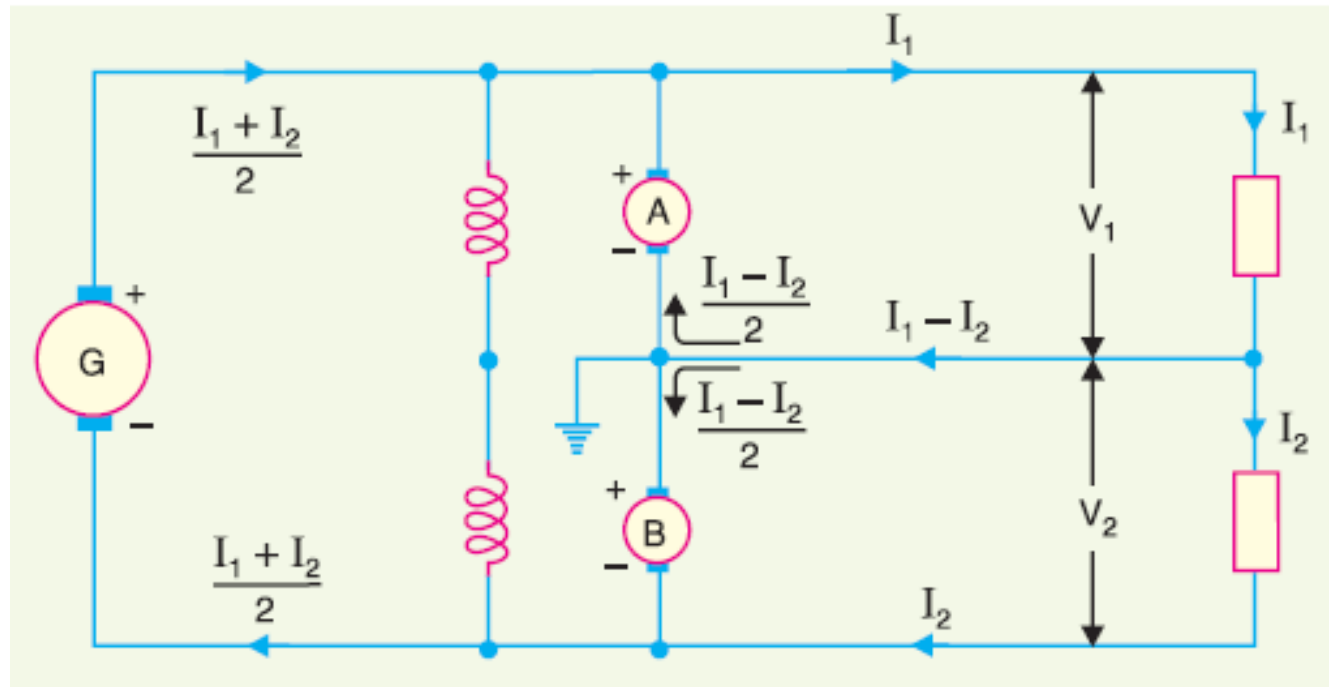
Voltage across load $DM = 249.17 - \text{drop in } CD - \text{drop in } ML + \text{drop in } KL = 249.17 - 0.3 - 0.18 + 0.084 = \mathbf{248.774 \text{ V}}$

Voltage across load $JG = 250 - \text{drop in } NJ - \text{drop in } GB = 250 - 0.2 - 1.2 = \mathbf{248.6 \text{ V}}$

Voltage across load $LH = 248.6 + \text{drop in } KJ - \text{drop in } KL - \text{drop in } HG = 248.6 + 0.28 - 0.084 - 0.864 = \mathbf{247.932 \text{ V}}$

Balancers in 3-Wire D.C. System

- In order to maintain voltages on the two sides of the neutral equal to each other, a *balancer set* is used.
- The balancer consists of two identical shunt wound machines **A** and **B** coupled mechanically and having their armature and field circuits connected in series across the outers.



Balancers in 3-Wire D.C. System

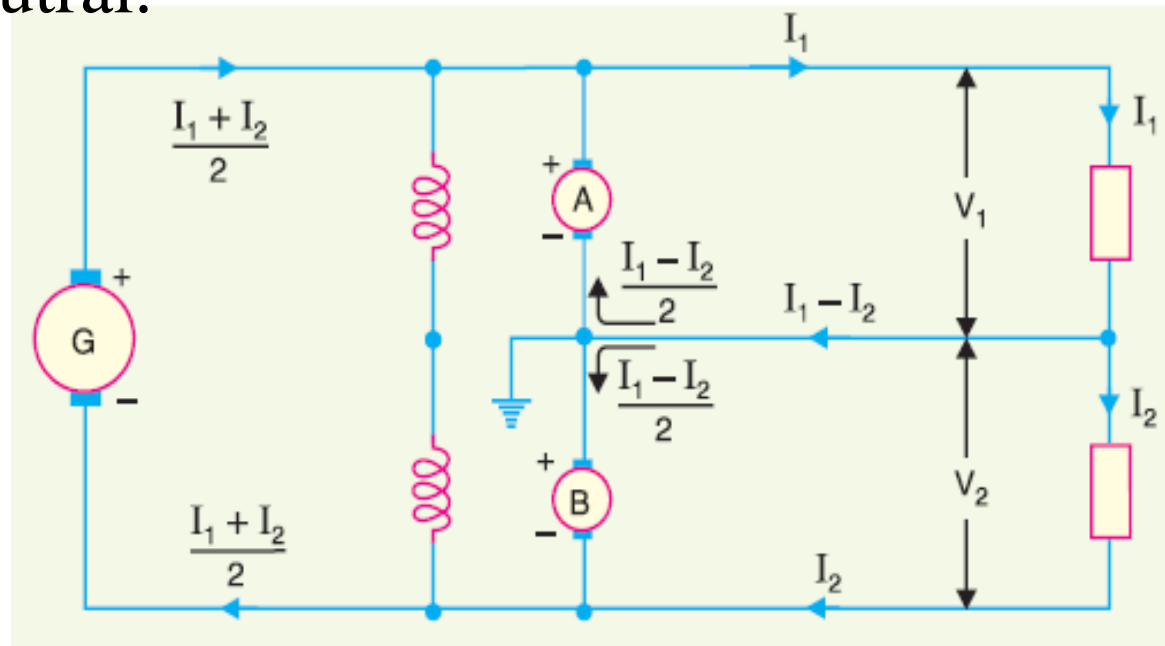
- The neutral wire is connected to the junction of the armatures.
- The circuit arrangement has two obvious advantages.
 1. Only one generator (G) is required which results in a great saving in cost.
 2. The balancer set tends to equalize the voltages on the two sides of the neutral.

$$E = V + I_a R_a$$

$$E > V \quad (G)$$

$$E = V \quad (\text{no load})$$

$$E < V \quad (M)$$



Balancers in 3-Wire D.C. System

Example 13.33. A d.c. 3-wire system with 500 V between outers has lighting loads of 150 kW on the positive side and 100 kW on the negative side. The loss in each balancer machine is 3 kW. Calculate :

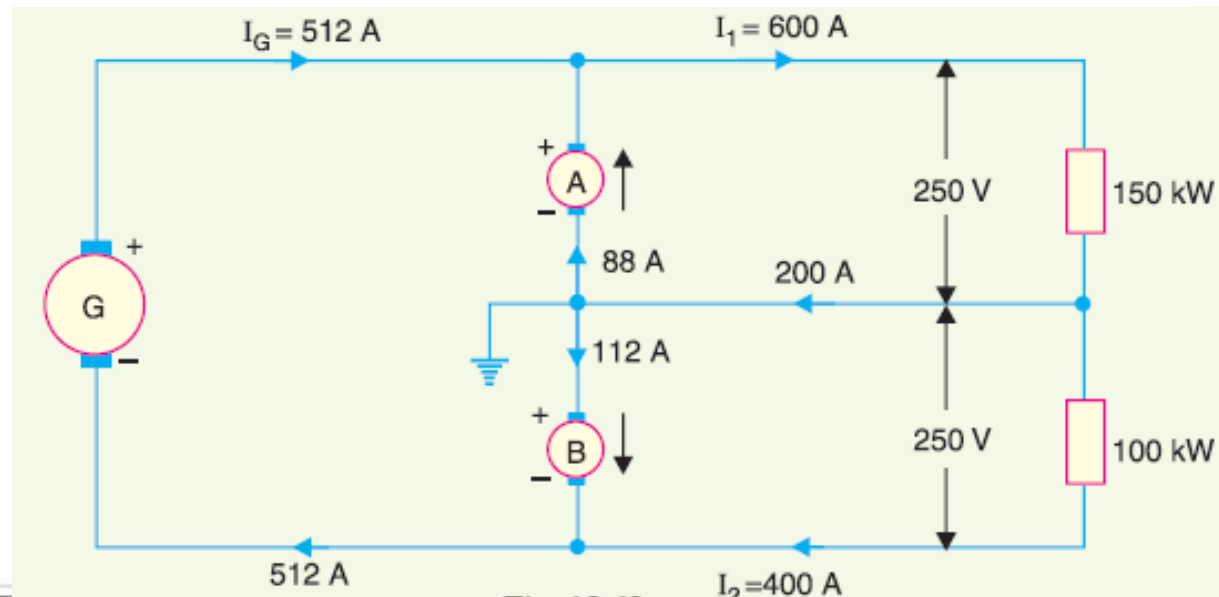
- (i) total load on the main generator
- (ii) kW loading of each balancer machine

Solution. The connections are shown in Fig. 13.62. As the positive side is more heavily loaded, therefore, machine *A* acts as a generator and machine *B* as a motor.

- (i) Total load on the main generator

$$= \text{load on +ve side} + \text{load on -ve side} + \text{losses}$$

$$= 150 + 100 + 2 \times 3 = \mathbf{256 \text{ kW}}$$



Balancers in 3-Wire D.C. System

(ii) Current supplied by the main generator,

$$I_G = 256 \times 10^3 / 500 = 512 \text{ A}$$

$$\text{Load current on +ve side, } I_1 = 150 \times 10^3 / 250 = 600 \text{ A}$$

$$\text{Load current on -ve side, } I_2 = 100 \times 10^3 / 250 = 400 \text{ A}$$

$$\text{Current in neutral wire} = I_1 - I_2 = 600 - 400 = 200 \text{ A}$$

$$\text{Current through machine } A = I_1 - I_G = 600 - 512 = 88 \text{ A}$$

$$\text{Current through machine } B = I_G - I_2 = 512 - 400 = 112 \text{ A}$$

$$\therefore \text{ Load on machine } A = 88 \times 250 / 1000 = \mathbf{22 \text{ kW}}$$

$$\text{Load on machine } B = 112 \times 250 / 1000 = \mathbf{28 \text{ kW}}$$